3.6 What is implicit differentiation?
3.7 How to use implicit differentiation to get an equation of *rates of change?*

2014-11-03 day 48

fire drill spot: 18C

3.6/37) $a^4 - t^4 = 6a^2 t$; find $\frac{da}{dt}$

we are finding the derivative of a with respect to t; in other words, a is a f^n of t. Think a(t).

$\frac{d}{dt}\left[a^4 - t^4 = 6a^2 t \right]$ (like x^4; $6a^2 t$ is a product)

CHAIN RULE (after power rule)

$$4a^3 \cdot \frac{da}{dt} - 4t^3 = 6\left[\frac{d}{dt}(a^2)\cdot t + a^2 \cdot \frac{d}{dt}(t)\right]$$

$$4a^3 \frac{da}{dt} - 4t^3 = 6\left[2a\frac{da}{dt}\cdot t + a^2\right]$$

$$4a^3 \frac{da}{dt} - 4t^3 = 12at\frac{da}{dt} + 6a^2$$

collect terms with $\frac{da}{dt}$ on one side, other stuff on other side

$$4a^3 \frac{da}{dt} - 12at\frac{da}{dt} = 4t^3 + 6a^2$$

$$\frac{da}{dt}(4a^3 - 12at) = 4t^3 + 6a^2$$

$$\frac{da}{dt} = \frac{(4t^3 + 6a^2)}{(4a^3 - 12at)} = \frac{2t^3 + 3a^2}{2a^3 - 6at}$$

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3.6 (23)

$x^3y^3 - 4 = 0$ (product)

Find $\frac{d^2y}{dx^2}$

first derivative

$\frac{d}{dx}(x^3)\cdot y^3 + x^3\cdot\frac{d}{dx}(y^3) - 0 = 0$

$3x^2\cdot y^3 + x^3\left[3y^2\cdot\frac{dy}{dx}\right] = 0$

$3x^3y^2\frac{dy}{dx} = -3x^2y^3$

$\frac{dy}{dx} = -\frac{3x^2y^3}{3x^3y^2} = -\frac{y}{x}$

second derivative

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = -\left[\frac{\frac{d}{dx}(y)\cdot x - y\cdot\frac{d}{dx}(x)}{x^2}\right]$

$= -\frac{x\frac{dy}{dx} - y}{x^2} = -\frac{x\left[-\frac{y}{x}\right] - y}{x^2}$

$= -\frac{-y-y}{x^2} = \frac{2y}{x^2}$

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3.6/41) Find a, b for $x^2y + ay^2 = b$ knowing $(1,1)$ is on graph

tangent line at $(1,1)$ is: $4x + 3y = 7$

[so slope $= \frac{dy}{dx} = -\frac{4}{3}$]

1) $1^2(1) + a(1)^2 = b$

$1 + a = b$

2) find $\frac{dy}{dx}$: $\left[\frac{d}{dx}(x^2)\cdot y + x^2\cdot\frac{d}{dx}(y)\right] + a\left[2y\frac{dy}{dx}\right] = 0$

$2xy + x^2\frac{dy}{dx} + 2ay\frac{dy}{dx} = 0$

$\frac{dy}{dx}(x^2 + 2ay) = -2xy$

$\frac{dy}{dx} = \frac{-2xy}{x^2 + 2ay}$ $\left.\frac{dy}{dx}\right|_{(x,y)=(1,1)} = \frac{-2}{1+2a} = -\frac{4}{3}$

so $-6 = -4(1+2a)$

$-2 = -8a$

$\frac{1}{4} = a$

$\frac{5}{4} = 1 + \frac{1}{4} = b$